Q:→ Solve $f(n+1) - f(n) = 3n^2 - n \quad , n \geq 0$

sol:→ Homo soln $f^h(n)$:

Associated Homo eqn $\quad f(n+1) - f(n) = 0$

For char eqn , take $f(n) = a^n$ in the above eqn

$$a^{n+1} - a^n = 0$$

$$\Rightarrow a - 1 = 0$$

$$\Rightarrow a = 1$$

$$\therefore f^h(n) = A(1)^n = A$$

Particular Soln $f^p(n)$:

$q(n) = 3n^2 - n$ which is a polyn of degree 2.

For particular soln , Take $f(n) = (d_0 + d_1 n + d_2 n^2) n$

$$= d_0 n + d_1 n^2 + d_2 n^3$$

in the given recc relation

$$f(n+1) - f(n) = 3n^2 - n$$

$$[d_0(n+1) + d_1(n+1)^2 + d_2(n+1)^3] - [d_0 n + d_1 n^2 + d_2 n^3] = 3n^2 - n$$

$$\Rightarrow [(n+1) - n] d_0 + [(n+1)^2 - n^2] d_1 + [(n+1)^3 - n^3] d_2 = 3n^2 - n$$

$$\Rightarrow d_0 + (n+1+n) d_1 + [(n+1)^2 + (n+1)n + n^2] d_2 = 3n^2 - n$$

$$\Rightarrow d_0 + (2n+1) d_1 + (3n^2 + 3n + 1) d_2 = 3n^2 - n$$

$$\Rightarrow (d_0 + d_1 + d_2) + (2d_1 + 3d_2) n + 3d_2 n^2 = 3n^2 - n$$

Equate coeff of n^0, n and n^2

$$d_0 + d_1 + d_2 = 0 \quad \Rightarrow d_0 = 1$$

$$2d_1 + 3d_2 = -1 \quad \Rightarrow d_1 = -2$$

$$3d_2 = 3 \quad \Rightarrow d_2 = 1$$

$$f^p(n) = d_0 n + d_1 n^2 + d_2 n^3 = n - 2n^2 + n^3$$

$$\therefore\ f^p(n) = d_0 n + d_1 n^2 + d_2 n^3 = n - 2n^2 + n^3$$
$$= n(1 - 2n + n^2) = n(n-1)^2$$

Complete solution

$$f(n) = f^h(n) + f^p(n)$$
$$= A + n(n-1)^2 \quad \underline{\text{Ans}}$$

Q:→ Form a recurrence relation satisfied by $a_n = \sum_{k=1}^{n} k^2$ and find value of $\sum_{k=1}^{n} k^2$.

sol:→

$$a_n = \sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \dots + n^2$$
$$a_{n-1} = \sum_{k=1}^{n-1} k^2 = 1^2 + 2^2 + \dots + (n-1)^2$$
$$\therefore\ a_n - a_{n-1} = n^2 \ ; \ a_1 = 1 \quad \text{——— (1)}$$

which is non homo linear recc. relation with const. coeffs.

<u>Homo soln (a_n^h) :—</u>

Associated Homo. eqn $\quad a_n - a_{n-1} = 0$ ——— (2)

For char eqn, take $a_n = a^n$ in (2)

$$a^n - a^{n-1} = 0$$
$$\Rightarrow a - 1 = 0$$
$$\Rightarrow a = 1$$
$$\therefore\ a_n^h = A(1)^n = A$$

<u>Particular soln (a_n^p) :→</u>

As $g(n) = n^2$, which is a polyn of degree 2.

For particular soln, take $a_n = (d_0 + d_1 n + d_2 n^2)\, n = d_0 n + d_1 n^2 + d_2 n^3$ in (1)

$$a_n - a_{n-1} = n^2$$
$$\Rightarrow [d_0 n + d_1 n^2 + d_2 n^3] - [d_0(n-1) + d_1(n-1)^2 + d_2(n-1)^3] = n^2$$

$$\Rightarrow [n-(n-1)]d_0 + [n^2-(n-1)^2]d_1 + [n^3-(n-1)^3]d_2 = n^2$$

$$\Rightarrow d_0 + (2n-1)d_1 + (3n^2-3n+1)d_2 = n^2$$

$$\Rightarrow (d_0-d_1+d_2) + (2d_1-3d_2)n + 3d_2n^2 = n^2$$

equate coeff of n^0, n and n^2

$$d_0-d_1+d_2 = 0 \quad \Rightarrow d_0 = 1/6$$

$$2d_1-3d_2 = 0 \quad \Rightarrow d_1 = 1/2$$

$$3d_2 = 1 \quad \Rightarrow d_2 = 1/3$$

$$\therefore \quad a_n^p = d_0n + d_1n^2 + d_2n^3 = \frac{n}{6} + \frac{n^2}{2} + \frac{n^3}{3} = \frac{n+3n^2+2n^3}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Complete solution

$$a_n = a_n^h + a_n^p$$

$$a_n = A + \frac{n(n+1)(2n+1)}{6}$$

As $a_1 = 1$

$$A + \frac{2\times 3}{6} = 1 \quad \Rightarrow \quad A = 0$$

$$\therefore \quad a_n = \frac{n(n+1)(2n+1)}{6}$$

$$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Q:→ Solve

$$S(n) - 4S(n-1) + 3S(n-2) = n^2$$

Ans $\quad A + B(3)^n - \frac{7}{3}n - n^2 - \frac{n^3}{6}$

Consider the recc relation

$$f(n) + c_1 f(n-1) + \dots + c_k f(n-k) = g(n)$$

Case IV :- $g(n) = Aq^n$ where q is const.

For particular soln, take $f(n) = dq^n$ where d is const. if q is not char root of the associated homo. eqn.

If q is char root of the associated homo. eqn then multiply the particular soln by n^m where m is the multiplicity of q in char roots.

Q:→ Solve

$$f(n) + 5f(n-1) + 6f(n-2) = 42(4)^n$$

sol:→ Homo soln $f^h(n)$:—

Associated Homo eqn $\quad f(n) + 5f(n-1) + 6f(n-2) = 0$

For char eqn, take $f(n) = a^n$ in above eqn

$$a^n + 5a^{n-1} + 6a^{n-2} = 0$$

$$\Rightarrow a^2 + 5a + 6 = 0$$

$$\Rightarrow (a+2)(a+3) = 0$$

$$\Rightarrow a = -2, -3$$

$$\therefore f^h(n) = A(-2)^n + B(-3)^n$$

Particular Soln $f^p(n)$

As $g(n) = 42(4)^n$; 4 is not char root

For particular soln, take $f(n) = q(4)^n$ in the given recc. relation

$$f(n) + 5f(n-1) + 6f(n-2) = 42\cdot(4)^n$$

$$\Rightarrow q(4)^n + 5q(4)^{n-1} + 6q(4)^{n-2} = 42\cdot(4)^n$$

$$\Rightarrow q(4)^2 + 5q(4) + 6q = 42(4)^2$$

$$\Rightarrow 16q + 20q + 6q = 42 \times 16$$

$$\Rightarrow 42q = 42 \times 16$$

$$\Rightarrow q = 16$$

$$\therefore f^p(n) = q(4)^n = 16\cdot(4)^n = 4^2\cdot(4)^n = 4^{n+2}$$

Complete soln

$$f(n) = f^h(n) + f^p(n)$$

$$= A(-2)^n + B(-3)^n + 4^{n+2}$$ Ans